

# *FORMA MEnTIS*: Learning Mathematics through an Interactive Web Teacher

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**Abstract.** *FORMA MEnTIS* is an interactive software tool able to encourage and support the diffusion and the dissemination of scientific culture through an innovative approach based on the integration of multimedia tools, Web and mathematical software. In this paper we will explore the main design and implementation issues of the *FORMA MEnTIS* project. In particular we will focus on tools to be used and how to use them in order to obtain a similar environment from scratch.

## 1. Introduction

In the last few years, the attention has turned on the use of the information and communication technologies for popular and didactical purposes. In particular, the great evolution of computers and computer networks has allowed the diffusion of multimedia applications for innovative presentation of knowledge and educational material.

Despite that, very often can be noted that such applications, exploiting new media and didactic supports, lack from the scientific-technological and contents presentation points of view. They are in many cases simple containers of information often not so well organised.

The realisation of multimedia applications for scientific dissemination has to foresee a careful analysis about contents and cognitive paths, putting particular attention to the introduction of meaningful interactive features. The latter can in fact increase the exemplification capabilities of the product and, consequently, the power of the dissemination process itself.

In this context has to be seen the research activity carried out within the *FORMA MEnTIS* project. Such project, in fact, aims to assist the scientific dissemination by creating a set of advanced services geared toward students and teachers of the secondary school.

*FORMA MEnTIS* is an Italian acronym that means: “advanced teaching through new methodologies and technologies in the school and in the society”. The final product of the *FORMA MEnTIS* project is a Web site providing a set of advanced services for the didactic of scientific disciplines, with particular attention to the Mathematics and the Computer Science.

Such site will be able to encourage and to support the diffusion of the scientific culture, through an innovative approach based on the integration of multimedia tools, Web and mathematical software as well as to establish a permanent interaction network between the world of secondary school and the world of university and research.

The purpose of this paper is to give a full description of the *FORMA MEnTIS* Web site both from the content and from the technological points of view.

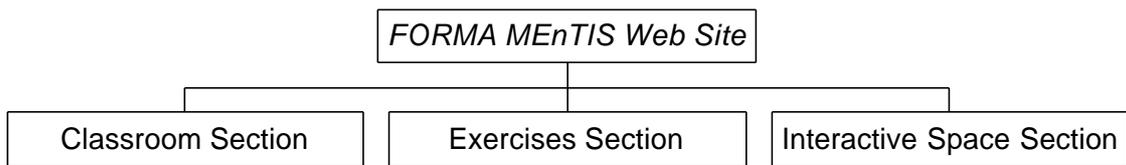


Figure 1. *FORMA MEnTIS* main structure.

## 2. The *FORMA MEnTIS* Web site

The *FORMA MEnTIS* Web site [1] is structured in three different main sections as it can be seen from the diagram in figure 1.

- The *Classroom* section gives learner the possibility to study (interactively with the contribution of Applets and JavaScript) theory about matrices and geometric transformations (as an example but extensible to cover many other topics) and some historical fundamentals about mathematics and computer science.
- The *Exercises* section is an interactive section in which students can exercise about learnt topics. In this section, using a server side mathematical engine, it is possible to generate exercises in real time, to test student using them and, if the student requires it, to show him the correct step-by-step solving procedure.
- The *Interactive Space* section gives users a set of services for interaction and co-operation: an on-line discussion forum where students can put questions and receive answers from teachers, a calendar of principal events, a test tools allowing teachers to send on-line tests to students and a graphical chat allowing teachers to do on-line lessons using texts an a virtual blackboard.

Following chapters are dealt with a detailed description of all Web site sections. In particular a double side view is given for each one of them. First of all the section is seen from the user point of view i.e. what the user sees and how he can exploit what he sees. After that, the developer point of view is discussed i.e. which technology is behind the section and how it can be carried out. Before starting the discussion about *FORMA MEnTIS* sections, some words must be spent around the adopted user interface [2] [3]. Figure 2a shows the home page of the site while figure 2b shows a generic site page (in particular that page comes from the *Classroom* section).



Figure 2 *FORMA MEnTIS* user interface: the home page (A) with its header (1) and a generic page from the courseware section (B) with its header (1) and its section topic menu (2).

As it can be seen, both pages (as all other *FORMA MEntIS* pages) have an header (1) showing the site logo and some buttons giving access to other sections. This gives, in every instant, the possibility to switch between sections avoiding disorientation as any feeling to be lost in the site.

Moreover, for sections composed by more then one page, a structured menu containing a line for each topic of the section (2 in figure 2b) is placed on the left of the user screen and the item corresponding to the currently displayed topic is highlighted. In this way the student has, in every instant, the precise notion of the position he reached in the lesson/page sequence and how many lessons/pages remain to be read to reach the end point.

Many times a student can encounter, during his learning activity, terms referring to an unknown or a forgotten concept. Using a little piece of JavaScript code, *FORMA MEntIS* pages are able to give to students the possibility to consult a sort of interactive glossary to discover or to remember the meaning of such terms (underlined in the text).

This glossary can't be implemented using the usual HTML link policy in order to avoid to distract user concentration from the current topic by substituting the actual displayed page. Our choice is to provide the explanation of such terms in a little window opened upon the current browser window. The student in this way can read the explanation without losing the current context (see figure 3).

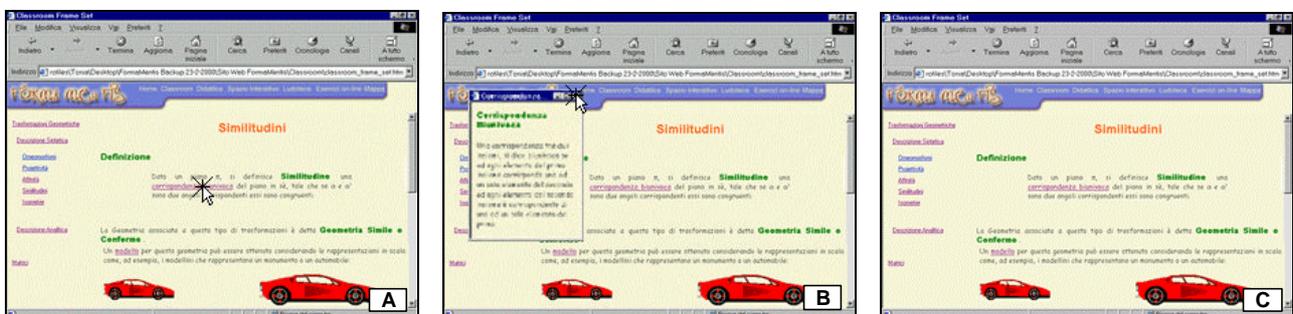
### 3. The Classroom Section

The *Classroom* Section is a small virtual classroom embedded in the *FORMA MEntIS* site. Here a student can deepen theory matters (in the subsection *Lessons of Mathematics*) or travel among Mathematics and Computer Science origins (in the subsections *Pages of history of Mathematics* and *Pages of history of Computer Science*). The following paragraph will provide a general overview of the whole section from the content organisation point of view (user view).

As we will see, the classroom section, in order to ensure a bigger level of understandability of treated concepts, makes a massive use of interactive Java applets. In the paragraph 3.2 we will explode such topic and, in particular, we will describe the *Transplet* package that we developed to manipulate geometric transformations inside a Web page (developer view).

#### 3.1. Content organisation (user view)

Figure 4 describes the structure of the whole *Classroom* section. As it can be seen, subsections are organised in several courses. At the present time, the *Lessons of Mathematics* subsection contains only two courses: *Plain Geometric Transformations* and *Matrices*.



**Figure 3.** *FORMA MEntIS* glossary: when the student clicks over an unknown term available in the glossary (A), a new window containing the requested explanation appears under the previous one (B). Once the student has read the explanation, he can close such window (C) and turn back to the lesson.

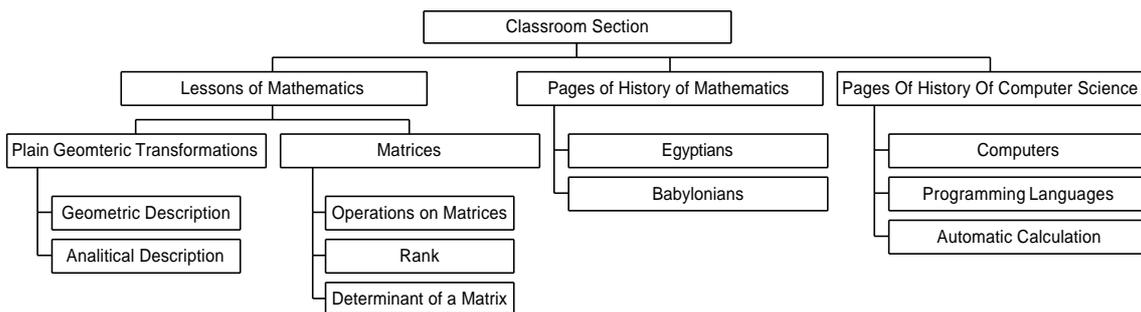


Figure 4. The Classroom Section structure

The *Plain Geometric Transformations* topic was chosen either cause the higher interest aroused among Secondary School teachers and because it is an interesting topic to be treated through interactive manipulation with geometric Java applets. The *Matrices* topic is intimately tied to geometric transformations so it was added to *FORMA MENTIS* courses.

As it can be seen still in figure 4, the *Plain Geometric Transformations* course [4] [5] can be studied from two different points of view: *Geometric* and *Analytical*. Along the *Geometric* pathway the learner can study Transformations starting from their geometric properties while, following the *Analytical* pathway, the learner can study the same topic starting from algebraic equations.

The didactical material has been structured in a hierarchical way in order to present the course on three deepening levels allowing to choose among three different didactical pathways. At the *first level* it is possible to give an intuitive general overview of the topic under examination with the aid of figures and applets. At the *second level* students can see a detailed list of properties defining formally the geometric transformation. Finally, at the *third level*, it is possible to obtain, for each property, the corresponding mathematical proof.

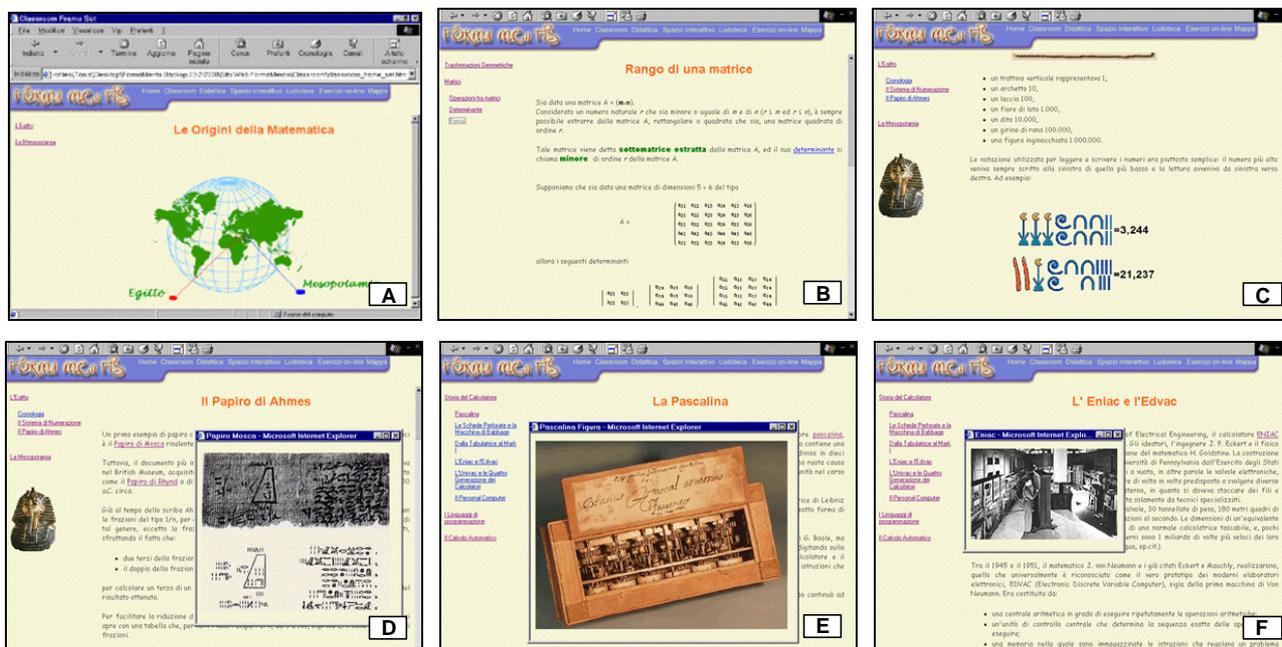


Figure 5. Selected images from Classroom Section: the selection of the ancient population to study (A), the explanation of the rank of a matrix (B), simple calculus made utilising Egyptian hieroglyphs (C), the ancient Moscow papyrus (D), the Pascalina (E), the Eniac (F).

As shown in figure 4, the *Matrices* course is organised in a general overview about the concept of matrix followed by a description of principal characteristics and operations on matrices. In this area too, many examples are shown, and an organisation in different deepening levels is provided.

If the student wants to travel among the Mathematics and Computer Science origins, he can access the *Pages of History of Mathematics* and *Pages of History of Computer Science* subsections.

In the *Pages of History of Mathematics*, [6] [7] the student can explore how the ancient, particularly the Egyptians and Babylonians, contributed to the growth of the mathematical disciplines. Such pages present many images showing ancient document and nice tools such as an interactive calculator utilising cuneiform number and Egyptian hieroglyphs rather than usual Arabic numbers. Figure 5 presents some selected pages from this section.

In the *Pages of History of Computer Science*, [8] [9] the fundamental steps of the discipline evolution are presented. The most meaningful moments of the computerised calculation growth are underlined. This subsection is divided in three areas which treat, respectively, *Computers*, *Programming Languages* and *Automatic Calculation* from its origins in the 1833, when Charles Babbage projected his Analytical Engine. Figure 5 presents some selected pages from this section.

### 3.2. *Transplet: a Java package for geometric transformations (developer view)*

As we seen, many images are distributed among *FORMA MEntIS* pages and many of them were made active through the use of Java applets. An applet is a mini-program that can run only under a Web browser. Applets are downloaded automatically as part of an HTML page (as any other component of the page i.e. graphic) and, when activated, they execute programs. [10]

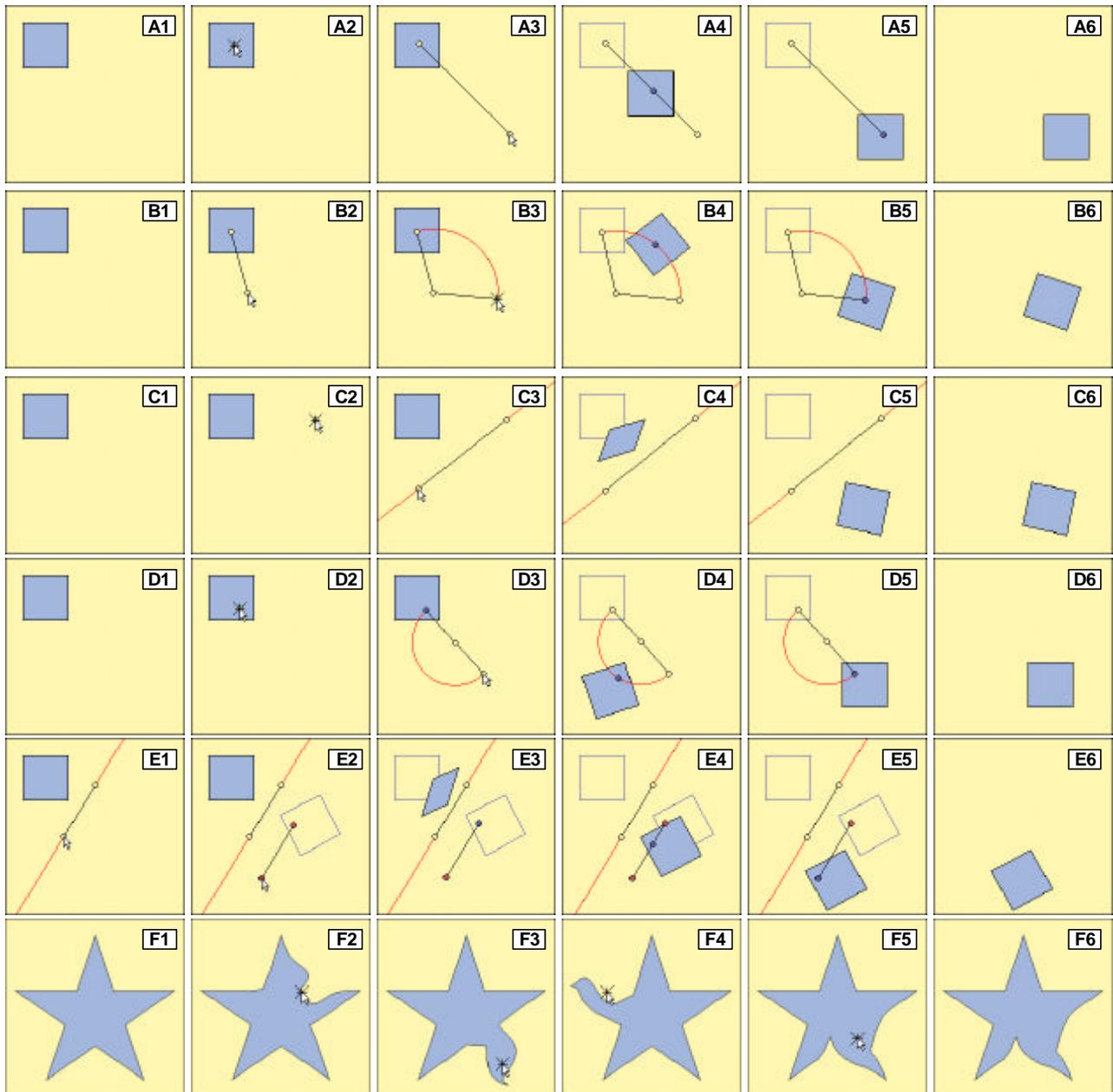
Applets are implemented using Java: a powerful programming language built to be secure, cross-platform and international. Differently from common programs, Java applets are able to be executed with all kind of computers having browsers with built-in Java interpreters.

Using applets is possible to let students interact with figures (that are truly applications) through simple mouse clicks. In such way the student is able to verify practically what learnt, applying intuitively the theoretical notions.

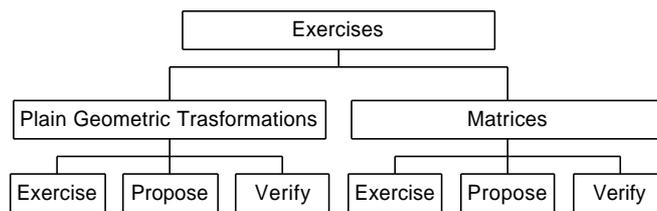
In order to apply these principles in the *FORMA MEntIS* context, we developed *Transplet*: a Java package for the didactic of the geometric transformations. *Transplet* contains six different applets referring to six different kinds of transformations:

- **Translation**: the student is able to translate a square (or any other shape) on a plane by tracing the directional vector simply dragging a point inside the square and dropping it in any place on the plane (figure 6a);
- **Rotation**: the student can rotate a square (or any other shape) on a plane by defining the rotation centre and angle, simply with mouse clicks and drags (figure 6b);
- **Axial Symmetry**: the student can obtain the symmetric image of a square (or any other shape) by drawing the axis of symmetry on the plane (figure 6c);
- **Central Symmetry**: the student can obtain the central symmetry of a square (or any other shape) as a rotation with fixed angle ( $180^\circ$ ) simply with a mouse point drags (figure 6d);
- **Glissosymmetry**: the student can obtain this kind of transformation on a square (or any other shape) by composing, with simple mouse clicks and drags, an axial symmetry and a translation whose directional vector is parallel to the axis of the symmetry (figure 6e);
- **Omeomorphism**: the student can obtain an omeomorphism of a star (or any other shape) by simply clicking on it in different places and observing how the shape will twist (figure 6f);

Such applets, embedded inside *FORMA MEntIS* pages, can be customised and used inside any HTML page. The *Transplet* package can in fact be downloaded from the *FORMA MEntIS* site and used by interested people free of charge [11] (comments are welcome).



**Figure 6.** *Transplet* from *FORMA MEntIS*. (A) **translation**: the student drags a point of the shape (2) and drops it in the plane (3) then the shape will move along the path (4,5); (B) **rotation**: the student sets an angle by dragging and dropping points (2,3) then the shape will move along the arc (4,5); (C) **axial symmetry**: the student draws the axis of the symmetry (2,3) then the shape will move three-dimensionally around such axis (4,5); (D) **central symmetry**: the student drags a point of the shape (2) and drops it in the plane (3) then the shape will move along the arc (4,5); (E) **glissosymmetry**: the student draws the axis of the symmetry (1) and the translation path from the projected shape (2) then the shape will move three-dimensionally around the axis (3) and along the path (4) from the projection point to the ending one (5); (F) **omeomorphism**: the student, clicking over the figure (2, 3, 4, 5), can obtain omeomorphisms of the beginning shape (6).



**Figure 7.** The Exercise section structure.

## 4. The Exercises Section

The *Exercises* section of the *FORMA MENTIS* site gives students the possibility to practice on the matters learnt in the *Lessons of Mathematics* section. The functions of this sections exploit a server side mathematical engine able to generate exercises in real time.

As the previous chapter, this chapter is composed by two paragraphs dedicated respectively to the user and the developer points of view. In the first paragraph, a general overview of the whole section is given, in the second one, implementation issues are discussed.

### 4.1. Exercises manipulation (user view)

Figure 7 describes the structure of the whole *Exercises* section. As it can be seen, there are two subsections, one for each course of the *Classroom* section. For each subsection, there are three different working modalities: *Exercise*, *Propose*, *Verify*.

Each working modality corresponds to a different way to exercise students. Utilising the *Exercise* modality, the learner, choosing a topic from the left menu of the HTML page, can propose to the system his own problem. After filling the form presented in the main page with the required set of problem parameters, he can ask the system to solve the problem.

As an example, if the student choose to do an exercise about similarities (see figure 8a), he must specify the vertex co-ordinates of the polygon to transform and the coefficients of the algebraic equations characterising the similarity. After doing that, by clicking on the **Solve** button, data are sent to the server side mathematical engine that will return a page containing not only the correct solution but also the graphical result of the transformation (see figure 8c).

Using the *Propose* modality, the user can chose a topic from the left menu and the system will return a new page containing a problem on the chosen topic followed by the correct solution that can be viewed only pressing the **Solve** button. The idea is that the student will solve the problem himself and compare his solution with the correct one presented by the system.



**Figure 8** Images from the Exercise section: data introduction in the Exercise (A) and in the Verify (B) modalities; graphical solution to an exercise (C).

Using the *Verify* modality, the learner has the possibility to verify the correctness of a his solution to a problem. He can choose a topic from the left menu and the system will show a page with a form to be filled with problem and solution parameters.

As an example, if the student choose to do an exercise about translations (see figure 8b), he must specify the vertex co-ordinates of the polygon to transform, the co-ordinates of the translation vector and the vertex co-ordinates of the transformed polygon. After doing that, by clicking on the **Verify** button, data are sent to the mathematical engine which will return a page containing the correctness of the given solution and a **Solve** button to be pressed if the student wants to view the correct solution.

#### 4.2. *Math Server: a server side mathematical engine (developer view)*

To realise the *FORMA MEnTIS Exercise* section, we interfaced a mathematical engine to the Web server managing the site. The mathematical engine we used is the *Mathematica 3.0<sup>TM</sup> Kernel*: a powerful application able to support numeric and symbolic calculation and to return graphics in two and three dimensions [12]. Exploiting such kernel installed on the server it is possible to make operations with the matrices and/or to apply geometric transformations to shapes easily by using the *Mathematica* language. In particular, two packages were created: one for each topic.

To interface such engine with the Web site we created *Math Server*: a server-side application based on CGIs [13]. CGI modules are applications able to extend a Web server with facilities for the dynamic generation of HTML pages. Client browsers are able to invoke CGI applications using the classical HTML POST action.

In our context the idea is the following: the student fills an exercise form with all needed parameters. After doing that, by pressing the **Solve** button, data will be posted to a CGI running on the server machine. Such CGI (that we named *Math Server*) is able to load a *Mathematica* package (associated with the action posted) and execute it (associating parameters collected from the client) communicating with the *Mathematica Kernel* running concurrently on the same machine. The *Math Server* and the *Mathematica Kernel* communicate through the *MathLink* protocol.

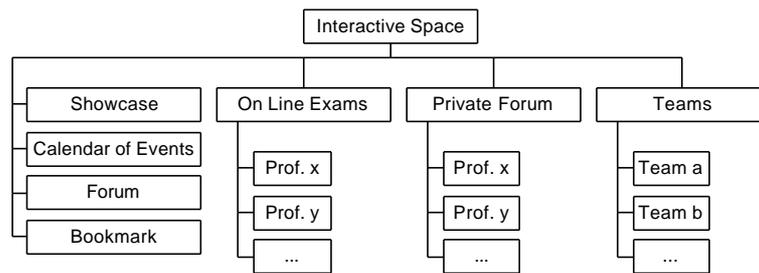
Once the results of the operation are calculated by the *Mathematica Kernel* they will return to the *Math Server* that will generate run-time an HTML page to be sent to the client browser. In other words, every connected user that executes an action inside the *Exercises* Section, makes a CGI process running on the Web Server. It receives Client requests and sends HTML pages to the Web server according to the computations of the *Mathematica Kernel*.

## 5. The Interactive Space Section

The *Interactive Space* section gives to students and teachers a set of services for interaction and co-operation (described in the following paragraph). In order to realise such services we used a commercial product named *SiteScape Forum*. A brief description of such application will be given in paragraph 5.2 (developer view).

### 5.1. *Interactive Space contents (user view)*

Figure 10a shows the first page of this section. The menu on the left side of the page contains a list of links which allow the access to all services. Any user can explore any service but, to fully exploit them, the user has to log in the system by providing a name and a password. This is required because users must be identified and the system has to know if they are common students or teachers.



**Figure 9.** The Interactive Space section structure.

To obtain user names and passwords it is necessary to register to the section by filling a form reachable from the left menu. The site administrator has the task to check registration requests and to add requiring people to the *Interactive Space*.

Figure 9 describes the structure of the whole *Interactive Space* section. As it can be seen, the *Interactive Space* is organised in several areas.

- **Showcase:** this area contains announces divided in several categories (find/change, movies, music, etc) accessible by all users but only registered users can add new announces;
- **Calendar of events:** an on-line calendar where it is possible to signal and retrieve events of the scholastic world (seminars, meetings school-family encounters, reunions, meetings, etc);
- **Forum:** in this area, it is possible to post questions, to attend answer, to open debates, to exchange opinions, suggestions, etc: some forums are just available, others can be added by registered users.
- **Bookmark:** in this area it is possible to find a lot of links to external sites about Mathematics and Computer Science. Any user can access to this area but only registered users can add new items. Links are organised in several groups respecting the *Classroom* section organisation.

To guarantee reliability of posted data, previous areas are moderated by the site administrator. When someone posts a message, this is firstly read by the administrator that, after realising its pertinence, releases the message by making it public. Following areas, instead, are organised as private forums.

- **Private Forum:** every teacher has his own forum where he can establish private conversations with his students. Students can require here deepening material, ask for explanations about understood topics, etc.
- **On Line Exams:** Every teacher has a private area where he can make exams. There are various examination modalities: multiple choice tests, questions, etc.



**Figure 10.** Images from the Interactive Section: the welcome page (A), a Forum page (B) and a team welcome page (C).

- **Teams:** all students of a class with their teachers form a team. Every team has its own area in which all member can share documents, have discussions about internal topics, have a bookmark list, a calendar of events, and so on.

## 5.2. The SiteScape Forum (developer view)

The *FORMA MEnTIS Interactive Space* section is realised using a commercial software named *SiteScape Forum* [14]. It is a team collaboration software providing an effective and easy way to communicate, share resources, and collaborate with groups of people. *Forum* users can host online discussions, share documents, participate in real-time chat, schedule meetings exploiting shared calendars; maintain personal homepages that include lists of favourite links, etc.

The difference between *SiteScape Forum* and many other collaboration software is its ability to be quickly and easily customised for any environment (like the *FORMA MEnTIS* one) by using the HTML template architecture. It is totally Web-based, requiring only an industry-standard browser to access, and runs on a broad range of platforms.

*Forum* also is useful in organising and finding information. An intuitive hierarchical folder system, as well as threaded documents and discussions, help keep information organised. Fully integrated AltaVista Search and Forum Tracker, which notifies users of new entries and keeps track of unseen entries, make finding information very easy.

## 6. Conclusions

In this paper we presented *FORMA MEnTIS*: an interactive tool able to encourage and support the diffusion and the dissemination of scientific culture through the integration of multimedia, Web and mathematical software. In particular we gave a double side view of *FORMA MEnTIS* components tacking into account users and developers needs. All main design and implementation issues involved in the creation of a similar site from scratch have been surveyed and many suggestions about product to use and useful literature have already been given.

It is important to note that *FORMA MEnTIS* is running on our Web server and can be reached at the URL specified in [1]. You can address any question and any comment about the site itself or about this paper to the site administrators (that are the authors of this paper too).

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